MATH 909 Continuum Mechanics-I

Credits: 3-0
Prerequisites: None

Course Objectives: This lecture course aims to introduce students to the basic concepts of Continuum Mechanics and linear elasticity

Core Contents: Tensors, basic constitutive laws of linear elasticity, stress and strain tensors in linear elasticity, elastic materials and symmetries, elasticity and problems related to reflection, refraction of waves, surface waves and wave guides.

Detailed Contents: Tensors: Definition of a tensor of order 2 and its extension to higher orders in arecursive manner. Change of basis. Covariant and contravariant tensors. Tensor algebra.
Symmetry in elastic materials: Periodicity in crystals, lattices, unit cell. The seven crystal systems. Effect of symmetry on tensors: Reduction of the number of independent components of a tensor dueto crystal symmetry, matrices for group symmetry elements in crystals, effect of a centre of symmetryand an axis of symmetry.
Static elasticity: The strain and stress tensors, equilibrium conditions. Hooke's Law. The elasticity tensor. Elastic energy in a deformed medium. Restrictions imposed by crystal symmetry on the number of independent elastic moduli.
Dynamic elasticity: Propagation equation, properties of elastic plane waves. Propagation along directions linked to symmetry. Elastic waves in an isotropic medium.
Reflection and refraction: Reflection of an SH wave from the surface of a half space. Reflection and refraction of a P-wave and and SV wave. Mode conversion.
Surface waves: The Rayleigh wave, uniqueness of the wave speed. The Love wave.
Wave guides: The Rayleigh Lamb dispersion relation for an isotropic plate. Lamb waves in an anisotropic plate.

Learning Outcomes: On successful completion of this course, students are expected to have:

- Understood mathematical definition of a tensor of rank n as a bilinear mapping from V^{n-1} to V,where V is a vector space. He/she should be adept at tensor algebra.
- Understood the symmetry groups associated with various classes of elastic materials.
- Understood equations of motion describing the dynamics of a continuum.
- Understood wave propagation in an anisotropic material.
- Understood the theory of Rayleigh waves, Love waves and Rayleigh-Lamb waves in awaveguide.
- Understood reflection and transmission of waves across an interface.

Text books

- ED: E. Dieulesaint and D. Royer, Elastic Waves in Solids-I, Free and Guided Waves, JohnWiley and Sons.(2000)

- JDA: J. D. Achenbach, Wave Propagation in Elastic Solids, North Holland.(1973)

Reference books

1. N.D. Critescu, E.M. Cracium and E. Soos, Mechanics of Elastic Components, Chapman and Hall.
2. T.C.T. Ting, Anisotropic Elasticity, Oxford University Press.

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown

Week	***Section***	***Topics***
1	Instructor's choice for book	Vector space, tensor of rank 2 as a linear mapping from V to V. Orthonormal bases.
2	-do-	Tensor of rank n. Tensor algebra.
3	ED 2.1-2.2	Symmetry in elastic materials, seven crystal systems.
4	ED 2.6	Reduction of number of independent components of a tensor due to symmetry.
5	ED 3.1	The strain and stress tensors. Physical interpretation of components. Equilibrium conditions.
6	ED 3.2	The elasticity tensor
7	ED 3.2	Restrictions imposed by crystal symmetry on the number of independent elastic moduli. Matrix representations for the seven crystal systems.
8	JDA 1.2	Linearzed theory of wave propagation, Waves in one dimensional longitudinal stress,
9	**Mid Semester Exam**	
10	JDA 2.4, 2.10	Elastic waves in an isotropic medium. The scalar and vector potentials.
11	JDA 4.1, 4.2	Plane waves, Time-harmonic plane waves
12	JDA 4.4 5.1-5.2, 5.4	Two dimensional wave motion with axial symmetry Joined half spaces
13	JDA 5.5-5.7	Reflection of an SH wave from the free surface of a half space. Reflection and transmission of a P wave and an SV wave, mode conversion.
14	JDA 5.11	The Rayleigh wave. Uniqueness of the phase speed
15	JDA 6.6	Propagation in a layer. Love wave.
16	JDA 6.7-6.8	Wave guides. The Rayleigh-Lamb dispersion relation in an isotropic plate. Analysis of the shape of the spectrum. The anomalous Lamb modes.
17		Review
18	**End semester Exam**	